Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 5SC02MTC2 **Subject Name: Partial Differential Equations**

Course Name: M.Sc. (Mathematics) Date: 20/5/2015

Semester: II Marks: 70

Time: 10:30 TO 01:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION – I

Q-1 (a) Solve: $r + 6s + 9t = 0$. [02]

(b) Classify region in which equation $y(x + y)(r - s) - xp - yq = 0$ is elliptic. [02]

(c) Write the equation into D & D' form: $2xr - 3pq^2 + rt - 5s^2 = 0$ [02]

(d) Find order and degree of the equation $5s^2 - 4r^3 = xyp$. [01]

Q-2 (a) a) Let $(\alpha D + \beta D' + \gamma)^n$ be a factor of $F(D, D')$ and $\alpha \neq 0$, then prove that [07]

$$u = e^{-\frac{\gamma x}{\alpha}} \sum_{s=1}^{n} x^{s-1}\, \phi_s(\beta x - \alpha y)$$

Is a solution of $F(D, D')z = 0$.

(b) Reduce the equation $r + 2s + t = 0$ to canonical form and hence solve it. [07]

OR

Q-2 (a) Solve: $(D^2 + DD' - 6D'^2)z = y\ \sin x$ [07]

(b) Classify and Reduce the equation $r = x^2 t$ to canonical form. [07]

Q-3 (a) Eliminate the arbitrary functions and obtained a partial differential equation [07]

$i)\ z = f(xy) + g\left(\frac{x}{y}\right)$

$ii)\ z = f(x + 5y) + g(x - 5y)$

(b) Solve:

$i)\ (D^3 - 3D^2D' + 2DD'^2)z = 0$ [07]

$ii) 25r - 40s + 16t = 0$

Q-3 (a) Prove that $F(D, D')[e^{ax+by} g(x, y)] = e^{ax+by}[F(D + a, D' + b]g(x, y)$, [07]

Where a and b are constants.

(b) Find the general solution of $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$. [07]

SECTION – II

Q-4 (a) Verify $u = x^2 - y^2$ is solution of two dimension Laplace equation. [02]

(b) Write the wave equation in cylindrical coordinate system. [02]

(c) Write Dirichlet problem for a Circle. [02]

(d) The poisson Integral formula can be obtained from Neumann BVP. Determine whether the statement is true or false. [01]

Q-5 (a) Solve interior Dirichlet problem for a function $\phi = \phi(r, \theta)$ for a circle and show that solution is of the form $\phi(r, \theta) = \sum_{n=0}^{\infty} r^n (A_n \cos n\theta + b_n \sin n\theta)$, With A_n, B_n constants. [07]

(b) Solve: $(x^2D^2 - y^2D'^2 + xD - yD')z = \log x$. [07]

OR

Q-5 (a) Using Monge's method, solve the equation $x^2r + 2xys + y^2t = 0$. [07]

(b) What is equipotential surface? Show that the surfaces $x^2 + y^2 + z^2 = c$ can form an equipotential family of surfaces, and find the general form of the potential function. [07]

Q-6 (a) Solve $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{k}\frac{\partial\psi}{\partial t} = 0$, by method of separation variable and show that solution is $\psi(x, y, z, t) = e^{\pm(lx+my+\gamma z+\alpha^2kt)}$ where l, m, γ, α are constant with $l^2 + m^2 + \gamma^2 = \alpha^2$. [07]

(b) Using Monge's method, solve the equation $3s - 2(rt - s^2) = 2$. [07]

OR

Q-6 (a) State and prove Harnack's theorem [07]

(b) Solve: $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = x^2y$ [07]